A Solid-State Microwave Source from Reactance-Diode Harmonic Generators*

T. M. HYLTIN†, MEMBER, IRE AND K. L. KOTZEBUE‡, MEMBER, IRE

Summary—**The generation of harmonics with reasonable efficiencies has been made possible by the application of high Q nonlinear reactance diodes. An approximate solution for the conversion loss of harmonic generators utilizing these devices has been obtained and design curves relating conversion loss with harmonic number, diode Q, and voltage-capacitance coefficient are presented. Harmonic generators have been operated with silicon and gallium-arsenide mesa diodes in the UHF region and conversion losses approaching the theoretical value have been obtained. Three harmonic generation stages in miniature modular packages were cascaded to obtained 7-mw output at S band. These stages were driven by a transistorized crystal-controlled oscillator and power amplifier which supplied 200 mw at 140 Mc from 1.3-w dc.**

Introduction

The generation of microwave frequencies at the milliwatt power level has in the past been the exclusive domain of klystrons and similar electron-beam devices. Unfortunately, in many satellite, missile, and portable ground equipment applications, the weight, size, reliability, and power consumption of these devices and their associated power supplies have proved to be a severe handicap. At lower frequencies these problems have often been solved by using semiconductor devices; for example, by replacing vacuum tubes with transistors. The upper frequency limit at which such transistor operation is practical is steadily increasing and is now approaching the microwave range.

Laboratory transistors have operated as oscillators at S band, and there are commercially available transistors capable of power output in the milliwatt range at 1000 Mc. As we go lower in frequency the power capabilities greatly increase; for example, power levels to 1 w are now available from transistors at as high as 150 Mc. Because of this relatively large power available in the VHF region, it is now possible to obtain useful power at microwave frequencies from an all-solid-state source by combining transistors with high-efficiency harmonic generators constructed from low-loss semiconductor diodes. This paper discusses the theoretical performance of harmonic generators and the design and performance of such a microwave source which weighs 13 ounces and is capable of an output power of 7 mw at 2520 Mc when connected to a 28-v, 1.3-w power source.

Theory of Harmonic Generation

Previous harmonic generators have made use of standard microwave mixer diodes. Page has shown that even an ideal nonlinear resistance generates harmonics with a conversion efficiency of $1/n^2$, where n is the desired harmonic.[1] A review of microwave components catalogs will show the realizable efficiency with presently available diodes to be an order of magnitude poorer than the $1/n^2$ expression indicates. It seems reasonable that harmonic generation by nonlinear reactance should have greatly enhanced efficiency, for a perfect reactance without dissipative loss can only transfer or store energy.

Indeed, it can be shown from the Manley-Rowe relations that the efficiency of generation of any harmonic can approach unity.[2] In practice, however, power flow at unwanted frequencies, and the inevitable loss of realizable circuit elements, will reduce that efficiency.[3,4]

An approximate solution for the conversion loss of a nonlinear reactance harmonic generator is not too difficult to obtain, provided we restrict ourselves sufficiently. Even though high efficiencies can be expected in many instances, we will assume the harmonic voltage to be small in order to keep the mathematics under control. Experimental results indicate that the error involved will normally be small in designs of practical interest. We will start with the fundamental equation for current:

$$i = \frac{dq}{dt} = \frac{dq}{dv}\frac{dv}{dt} = c(v)\frac{dv}{dt}. \tag{1}$$

We will assume that the ac voltage is composed of a large fundamental voltage, v_1, and small harmonic voltages, v_2. With this assumption we can expand the capacitance function, $c(v)$, in a Taylor series about v_1:

$$c(v) = c(v_1) + \left.\frac{dc}{dv}\right|_{v=v_1} v_2 + \cdots \tag{2}$$

We can also write the identity

$$\frac{dc}{dt} = \frac{dc}{dv}\frac{dv}{dt}; \tag{3}$$

[1] C. H. Page, "Harmonic generation with ideal rectifiers," Proc. IRE, vol. 46, pp. 1738–1740; October, 1958.

[2] J. M. Manley and H. E. Rowe, "Some general properties of nonlinear elements," Proc. IRE, vol. 44, pp. 904–913; July, 1956.

[3] D. Leenov and A. Uhlir, "Generation of harmonics and subharmonics at microwave frequencies with p-n junction diodes," Proc. IRE, vol. 47, pp. 1724–1729; October, 1959.

[4] D. Leeson and S. Weinreb, "Frequency multiplication with nonlinear capacitors—a circuit analysis," Proc. IRE, vol. 47, pp. 2076–2084; December, 1959.

* Received by the PGMTT, July 26, 1960; revised manuscript received, September 26, 1960.

† Texas Instruments, Inc., Dallas, Tex.
‡ Watkins-Johnson Co., Palo Alto, Calif.

therefore,

$$\frac{dc}{dv}_{v=v_1} = \frac{\dfrac{dc}{\left(\dfrac{dc}{dt}\right)_{v=v_1}}}{\dfrac{dv}{\left(\dfrac{dc}{dt}\right)_{v=v_1}}}. \qquad (4)$$

Substituting (4) and (2) into (1), we get

$$i = c(v_1)\left[\frac{dv_1}{dt} + \frac{dv_2}{dt}\right] + \frac{dc(v_1)}{dt} v_2 \left[1 + \frac{\dfrac{dv_2}{dt}}{\dfrac{dv_1}{dt}}\right] + \cdots \qquad (5)$$

To obtain linear equations, we assume that

$$\frac{\dfrac{dv_2}{dt}}{\dfrac{dv_1}{dt}}$$

is small compared to unity. With this approximation, the expression for current becomes

$$i = c(v_1)\left[\frac{dv_1}{dt} + \frac{dv_2}{dt}\right] + \frac{dc(v_1)}{dt} v_2. \qquad (6)$$

In the evaluation of (6) we will use exponential representations. Let

$$v_1 = V_1 e^{j\omega_1 t} + V_1^* e^{-j\omega_1 t}, \qquad V_1 = V_1^*$$

$$v_2 = \sum_{\substack{n=-\infty \\ \neq \pm 1}}^{\infty} V_n e^{jn\omega_1 t}$$

$$c(v_1) = C_0 \sum_{n=-\infty}^{\infty} \gamma_n e^{jn\omega_1 t} \qquad \gamma_n = \gamma_{-n}$$

$$i = \sum_{n=-\infty}^{\infty} I_n e^{jn\omega_1 t}. \qquad (7)$$

Substituting these values into (6), we obtain the following expression for the nth harmonic current:

$$I_n = j\omega C_0[\gamma_{n-1} - \gamma_{n+1}]V_1 + jn\omega C_0 \sum_{\substack{k=-\infty \\ \neq \pm 1}}^{\infty} \gamma_{n-k} V_k. \qquad (8)$$

From this equation we can construct an admittance matrix which will describe the "small-signal" harmonic generator. Before carrying the analysis further, let us assume that our circuit is so constructed as to allow significant voltages to exist across the capacitor only at the fundamental and nth harmonic. This is a reasonable assumption for many practical circuits, since the diodes will be of relatively high Q. With only V_1 and V_n not equal to zero, we can then obtain from (8) the following equations for I_1 and I_n:

$$I_1 = j\omega C_0[1 - \gamma_2]V_1 + j\omega C_0 \gamma_{n-1} V_n$$
$$I_n = j\omega C_0[\gamma_{n-1} - \gamma_{n+1}]V_1 + jn\omega C_0 V_n. \qquad (9)$$

In matrix notation we have,

$$\begin{bmatrix} I_1 \\ I_n \end{bmatrix} = j\omega C_0 \begin{bmatrix} 1 - \gamma_2 & \gamma_{n-1} \\ \gamma_{n-1} - \gamma_{n+1} & n \end{bmatrix} \begin{bmatrix} V_1 \\ V_n \end{bmatrix}. \qquad (10)$$

This matrix represents the admittance of the lossless, small-signal harmonic generator. We will approximate the effect of series loss by inverting the admittance matrix and by adding a series resistance. The *lossless* inverted matrix equation is:

$$\begin{bmatrix} V_1 \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_n \end{bmatrix}, \qquad (11)$$

where

$$Z_{11} = \frac{n}{j\omega C_0 D}$$

$$Z_{12} = -\frac{\gamma_{n-1}}{j\omega C_0 D}$$

$$Z_{21} = -\frac{\gamma_{n-1} - \gamma_{n+1}}{j\omega C_0 D}$$

$$Z_{22} = \frac{1 - \gamma_2}{j\omega C_0 D}$$

$$D = n(1 - \gamma_2) - \gamma_{n-1}(\gamma_{n-1} - \gamma_{n+1}) \approx n(1 - \gamma_2).$$

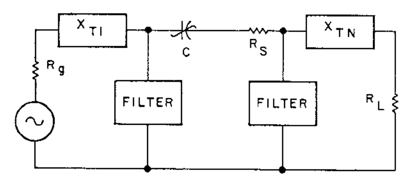


Fig. 1—Circuit model used in the analysis of harmonic generation.

The circuit model used in the derivation of gain (actually a loss) is shown in Fig. 1. We can write the transducer gain for this circuit as

$$g_T = \frac{4R_g R_L |Z_{21}|^2}{|(Z_{11} + Z_{T1})(Z_{22} + Z_{Tn}) - Z_{12}Z_{21}|^2}. \qquad (12)$$

Substituting the matrix values we have,

$$g_T = \frac{4R_g R_L \left(\dfrac{\gamma_{n-1} - \gamma_{n+1}}{1 - \gamma_2}\right)^2 \dfrac{1}{(n\omega C_0)^2}}{\left|\left(\dfrac{1}{j\omega C_0(1 - \gamma_2)} + Z_{T1}\right)\left(\dfrac{1}{j\omega n C_0} + Z_{Tn}\right) + \dfrac{\gamma_{n-1}(\gamma_{n-1} - \gamma_{n+1})}{(1 - \gamma_2)^2} \dfrac{1}{(n\omega C_0)^2}\right|^2}. \qquad (13)$$

Now, let us assume that we impose the tuning conditions

$$\text{Imag } X_{+1} = \frac{1}{\omega C_0(1-\gamma_2)},$$

and

$$\text{Imag } X_{Tn} = \frac{1}{n\omega C_0}. \tag{14}$$

Further, let us assume that all circuit loss, exclusive of source or load resistance, is series loss associated with the nonlinear capacitor. Therefore, $R_{T1}=R_g+R_s$ and $R_{Tn}=R_L+R_s$. Eq. (13) then becomes

$$g_T = \frac{4R_gR_L\left(\frac{\gamma_{n-1}-\gamma_{n+1}}{1-\gamma_2}\right)^2 \frac{1}{(n\omega C_0)^2}}{\left[(R_g+R_s)(R_L+R_s)+\frac{\gamma_{n-1}(\gamma_{n-1}-\gamma_{n+1})}{(1-\gamma_2)^2}\frac{1}{(n\omega C_0)^2}\right]^2}. \tag{15}$$

We can maximize the gain by choosing the proper values for the source and load resistances. A little calculus will show the values to be

$$R_g = R_L = R_s\sqrt{1+\bar{\gamma}^2Q_n^2} \tag{16}$$

where

$$\bar{\gamma}^2 = \frac{\gamma_{n-1}(\gamma_{n-1}-\gamma_{n+1})}{(1-\gamma_2)^2}$$

and

$$Q_n = \frac{1}{n\omega C_0 R_s}.$$

Substituting this optimum value into the expression for transducer gain we get, after some manipulation,

$$g_T = \frac{\gamma_{n-1}-\gamma_{n+1}}{\gamma_{n-1}} \frac{\bar{\gamma}^2Q_n^2}{[1+\sqrt{1+\bar{\gamma}^2Q_n^2}]^2}. \tag{17}$$

Notice that as we allow the loss to approach zero (by making Q large), the transducer gain approaches unity. That it will never quite reach unity may be attributed to our initial small-signal approximations.

Using computed values for the capacitance Fourier coefficients, we can readily obtain curves which show how much loss can be expected in harmonic generators using semiconductor diodes. Several such sets of curves are shown in Figs. 2–7 (next page). Here we have plotted conversion loss in db as a function of the diode Q at the fundamental frequency for the second, third, fourth, sixth, and eighth harmonics. Figs. 2, 3, and 4 show results applicable to diodes which have $1/\sqrt{V}$ capacitance variations (such as the alloy-junction diodes), while Figs. 5, 6, and 7 show results applicable to $1/\sqrt[3]{V}$ capacitance variations (such as the diffused-junction diodes). The parameter, a, is the ratio of ac voltage amplitude to dc bias (including contact potential). The proper value of a cannot at this time be precisely stated. A combination of theory and experiment indicates that for peak ac voltages of around 5 volts or less, a value of $a=0.90$ is reasonable. For lower or higher drive levels the insertion-loss curves corresponding to $a=0.85$ and $a=0.95$ are included as well as those for $a=0.90$.

If we follow the change in conversion efficiency as a function of drive level, we will observe that at high efficiencies the curves predict saturation phenomena. For example, Fig. 2 indicates that doubling with a diode of $Q=100$ will yield a conversion loss of 1.6 db, while Fig. 4 indicates a loss of 2.1 db. Although such a saturation characteristic is actually observed in practice[5,6] it is likely that the saturation effect predicted by the curves is actually due to errors introduced by the original small-signal approximation. The saturation effects encountered experimentally can probably be attributed to a decrease in the effective Q of the diode caused by forward or reverse conduction.

Several conclusions may be drawn from Figs. 2 through 7. One is that for efficient generation, particularly at the higher harmonics, diode and circuit Q's must be quite high. It should also be noted that the generation of higher harmonics with this type of circuit is best accomplished in stages. As an example, let us assume that we wish to generate the sixth harmonic with a diode of $Q=100$ at the fundamental frequency in circuits having negligible losses. Fig. 3 shows that if we go directly to the sixth harmonic we should expect a conversion loss of 15.5 db. If we choose to double and then to triple with two diodes of the same quality as used in the previous example, we find a conversion loss of 1.8 db in doubling, and 4.5 db in tripling ($Q=50$ for the tripler), which gives a total conversion loss of only 6.3 db, an improvement of 9.2 db over the single-stage conversion efficiency. If we triple first and then double, we get even better efficiency: only a 5.4-db conversion loss. This is one order of magnitude improvement over the single-stage conversion efficiency.

The example above used the curves corresponding to abrupt-junction diodes for which the capacitance variation is proportional to $1/\sqrt{V}$. For a linearly-graded junction, those with a capacitance proportional to $1/\sqrt[3]{V}$, the predicted efficiencies are lower. Direct conversion to the sixth harmonic, as in the above example with a linearly-graded diode, will result in a 22-db loss, 6.5 db below the output with a comparable abrupt

[5] R. Lowell and M. J. Kiss, "Solid-state microwave power sources using harmonic generation," Proc. IRE, vol. 48, pp. 1334–1335; July, 1960.

[6] D. Leenov and J. W. Rood, "UHF harmonic generation with silicon diodes," Proc. IRE, vol. 48, p. 1335; July, 1960.

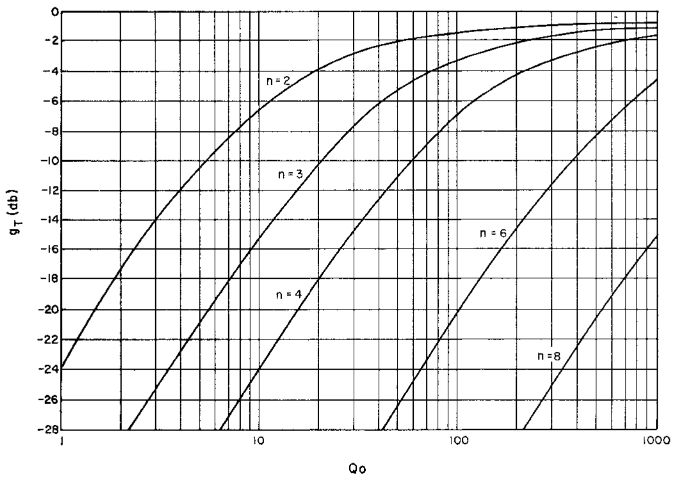


Fig. 2—Computed conversion loss of harmonic generation as a function of diode Q at the fundamental frequency. $a=0.85$. Inverse square-root capacitance variation with voltage.

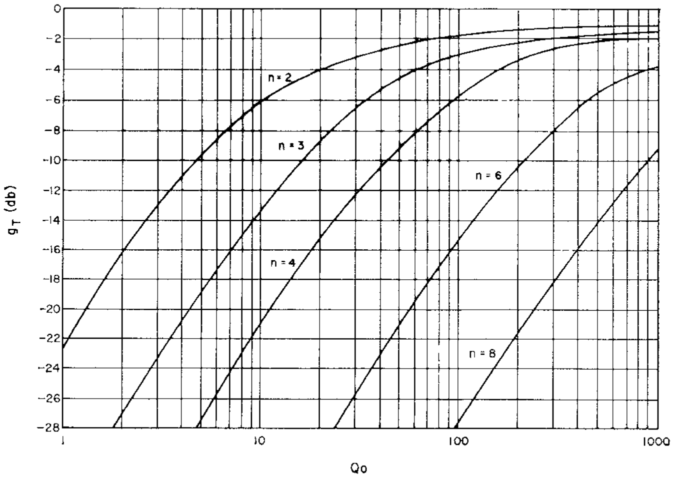


Fig. 3—Computed conversion loss of harmonic generation as a function of diode Q at the fundamental frequency. $a=0.90$. Inverse square-root capacitance variation with voltage.

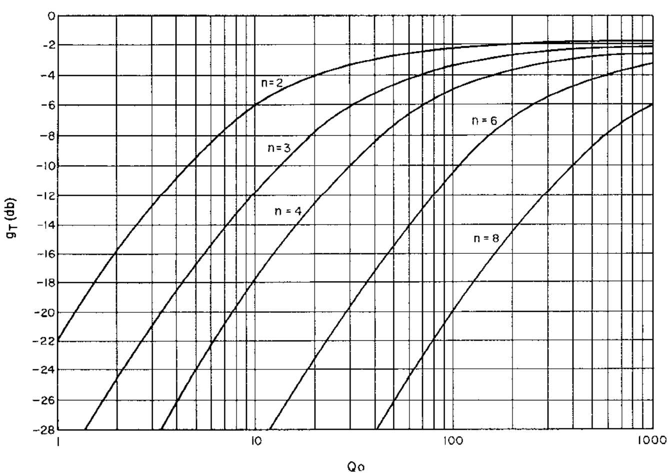


Fig. 4—Computed conversion loss of harmonic generation as a function of diode Q at the fundamental frequency. $a=0.95$. Inverse square-root capacitance variation with voltage.

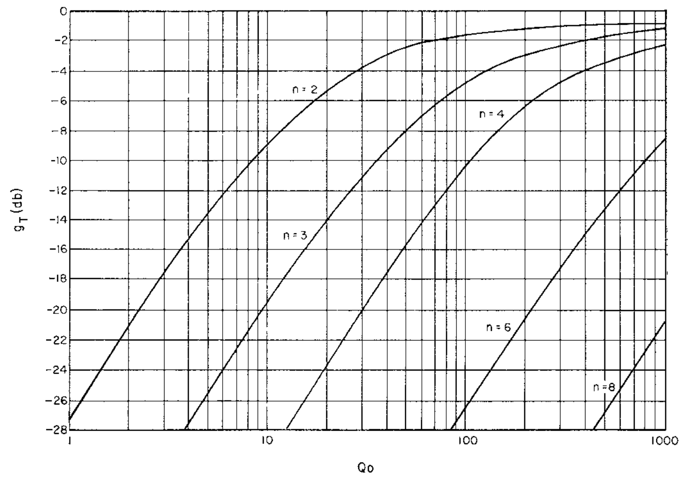


Fig. 5—Computed conversion loss of harmonic generation as a function of diode Q at the fundamental frequency. $a=0.85$. Inverse cube-root capacitance variation with voltage.

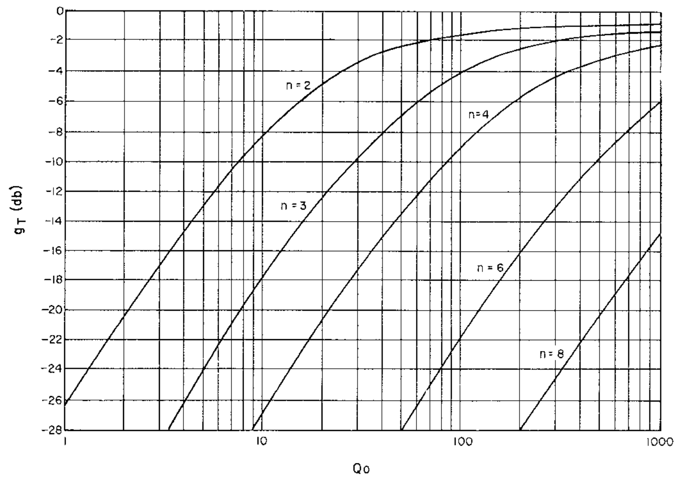


Fig. 6—Computed conversion loss of harmonic generation as a function of diode Q at the fundamental frequency. $a=0.90$. Inverse cube-root capacitance variation with voltage.

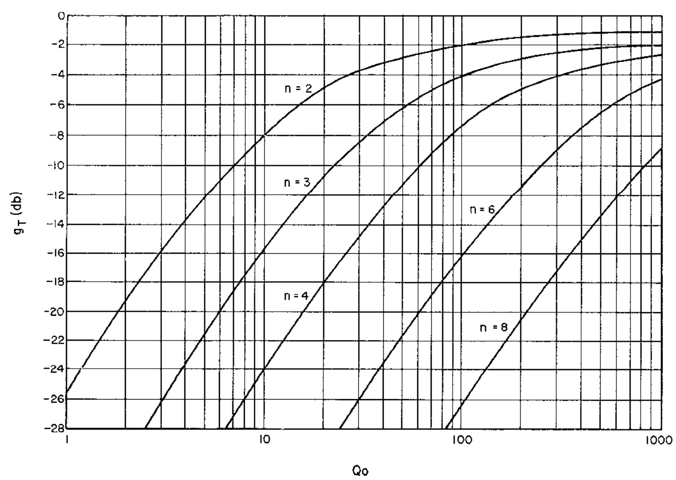


Fig. 7—Computed conversion loss of harmonic generation as a function of diode Q at the fundamental frequency. $a=0.95$. Inverse cube-root capacitance variation with voltage.

junction. A tripler, followed by a doubler, will have a combined loss of 7.4 db, a 2-db greater loss than the previous example using abrupt junction diodes. At the present time, though, diffused silicon diodes are available with a greater reverse breakdown voltage than diodes having abrupt junction characteristics and, therefore, have greater power handling capabilities.

Experimental Results

To apply harmonic generation to a specific task, considerable attention must be given to the choice of input frequency and to the number and type of multiplier stages. When designing for maximum power output at some desired frequency these factors will be determined by the quality and capabilities of the transistors and reactance diodes available at the time of design. This design procedure may best be illustrated by describing a crystal-controlled local oscillator source that supplies 5 to 10 mw of power at 2520 Mc.

To best utilize the reactance diodes available at the time of design, the sequence of multiplication chosen was: triple from 140 to 420 Mc, double from 420 to 840 Mc, and triple again from 840 to 2520 Mc. The last tripler stage was justified by the high-quality reactance diodes available for use at low power levels. A crystal-controlled oscillator, a doubler, and a power amplifier was designed using three 2N1141 transistors which had an output of 200 mw at 140 mc. This circuit, shown in Fig. 8, required 1.3 watts input at 28 volts.

The application of harmonic generation to many microwave problems depends upon the successful packaging of these circuits. A miniature modular construction was used which is light and rugged, yet retains the flexibility of cascading stages to achieve various frequencies and power from common modules. The transistor crystal-controlled oscillator, doubler, and power amplifier were readily adapted to this modular construction. A circuit for the tripler from 140 to 420 Mc is suggested by the circuit model used in the theoretical study (Fig. 1). The actual circuit used is shown in Fig. 9(a). By using miniature piston capacitors and a subminiature potentiometer in this tripler circuit, the size was reduced for packaging in a one cubic inch module. An input of 200 mw at 140

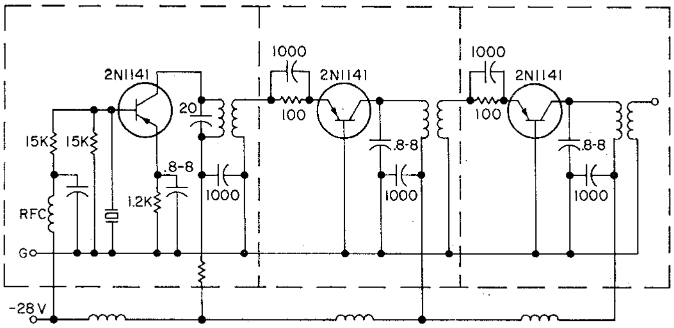
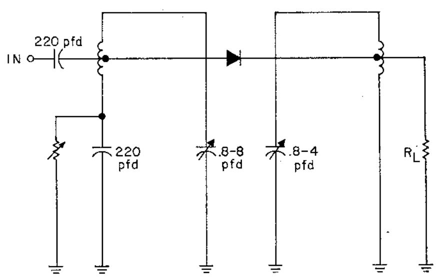
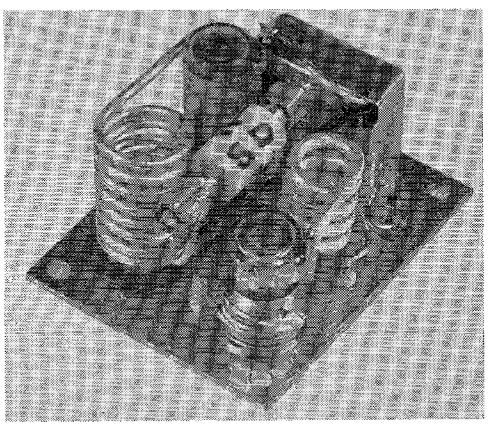


Fig. 8—Schematic diagram of the transistorized crystal-controlled oscillator, doubler, and power amplifier.

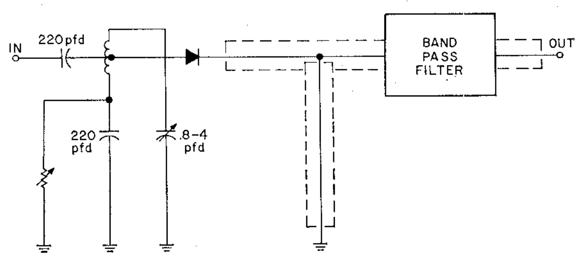


(a)

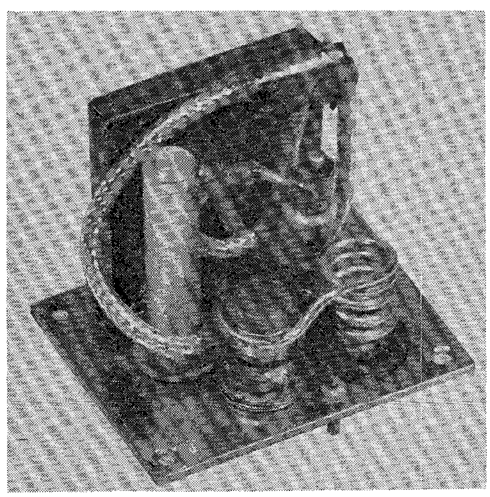


(b)

Fig. 9—(a) Schematic diagram of the reactance-diode tripler from 140 Mc to 420 Mc. (b) Photograph of the reactance-diode tripler from 140 Mc to 420 Mc.



(a)



(b)

Fig. 10—(a) Schematic diagram of the reactance-diode doubler from 420 Mc to 840 Mc. (b) Photograph of the reactance-diode doubler from 420 Mc to 840 Mc.

Mc produced 80 mw output at 420 Mc. All spurious outputs were suppressed a minimum of 15 db. This 4-db conversion loss was obtained with a diode having a 4-picofarad capacitance at 2-v reverse bias, and a 2.5-ohm series resistance. As reverse breakdown of this diode occurred at approximately 80 volts, considerably more power could be handled by this tripler.

For the doubler to 840 Mc the circuit shown in Fig. 10(a) was evolved which consisted of both lumped and distributed elements. Circuit elements as used in the preceding tripler formed the input of the doubler and miniature coaxial components were developed for use in the output. A stub tuner 3/16-inch in diameter and 7/8-inch long was employed for output tuning and a "Triplate" filter 0.8 inch wide, 0.9 inch high and 1/8 inch thick provided output filtering. A diffused silicon diode with a 0.95-picofarad junction capacitance and 9-ohm series resistance was used in this circuit as shown assembled in Fig. 10(b). An input of 80 mw at 420 Mc produced an output of 30 mw at 840 Mc.

The final tripler to 2520 Mc was designed with distributed circuits at both input and output as shown in Fig. 11. This was reduced to the required 0.8 inch by 0.9 inch by 1 inch size by using Microdot cable to form the resonant lines and a subminiature teflon-loaded filter as the output filter. Approximately 7 mw output was obtained with 30 mw input for a 6.3-db conversion loss. This is 3 db below the theoretical output associated with a 45-kMc cutoff diode. One and one-half-db loss is due to the output filter, and the remaining loss is attributed to losses in the Microdot cable and to tuning problems. The assembled S-band source is shown in Fig. 12.

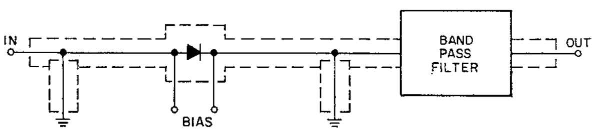


Fig. 11—Schematic diagram of the reactance-diode tripler from 840 Mc to 2520 Mc.

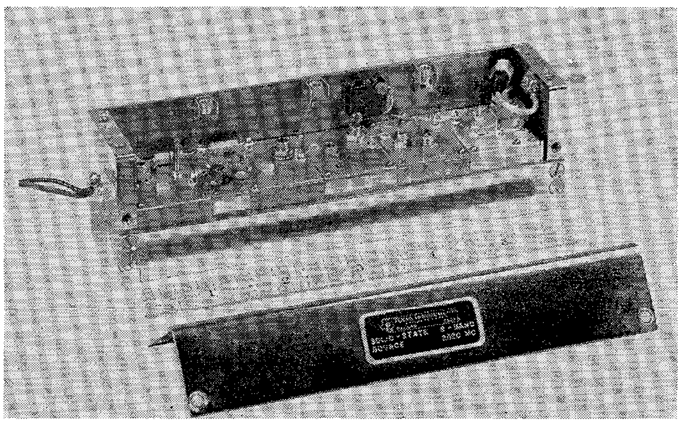


Fig. 12—Photograph of assembled solid-state S-band signal source.

Conclusion

It is now possible to design a crystal-controlled local oscillator source at microwave frequencies which is light, rugged, and requires only about 1/10 the input power required by other means of local oscillator generation. There are several operating characteristics that should be noted. Since harmonic generators operate with relatively constant efficiency from very low power up to saturation, a lower output power may be accompanied by a corresponding reduction in input power. The power supply problem is greatly simplified, both by the elimination of the heater supply and by the self-limiting characteristic of the harmonic generator as shown in Fig. 13. A 10-per cent deviation from the 28-volt input will cause less than 1-db output variation. This should permit the operation of such microwave sources from unregulated supplies, solar cells, or batteries.

The harmonic generation stages may be considered to be a series of high-Q coupled resonant circuits, and as such, the band-pass is very narrow. Future research might lead to broad-banding techniques but, at present, cascaded harmonic generators have only sufficient band-pass for simple AFC.

Harmonic generation to higher frequencies is limited only by the quality of the reactance diodes. With diodes having a cutoff frequency in the range of 100 to 200 kMc, solid-state power sources to K_a band should now be possible.

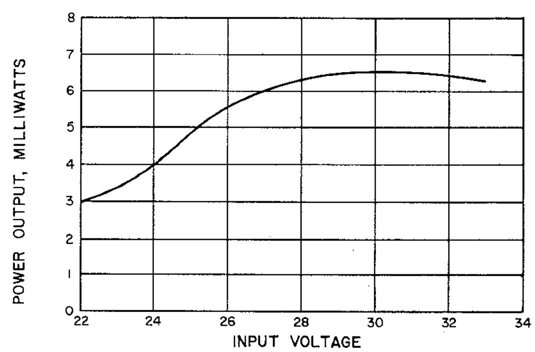


Fig. 13—Plot of the measured of RF power output at 2520 Mc as a function of the applied direct voltage.